Iteration Bound

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Outline

- Introduction
- Data Flow Graph (DFG) Representations
- Loop Bound and Iteration Bound
  - Longest Path Matrix Algorithm (LPM)
  - Minimum Cycle Mean Method (MCM)
- Conclusion
Introduction

Many DSP algorithms such as recursive and adaptive digital filters contain feedback loops, which impose an inherent fundamental lower bound on the achievable iteration or sample period.

- Iteration bound = maximum loop bound
- Clock period = critical path period = cycle time = 1/clock rate
- Sample rate = throughput rate
- Impossible to achieve an iteration bound less than the theoretical iteration bound with infinite processors
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- **Loop Bound and Iteration Bound**
- Compute the Iteration Bound
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DSP Program: Example

for \( n = 0 \) to \( \infty \)

\[ y(n) = ay(n-1) + x(n) \]

Intra - Iteration Period: \( A_k \rightarrow B_k \)

Inter - Iteration Period: \( B_k \Rightarrow A_{k+1} \)

Data Flow Graph
Loop Bounds in DFG

- Critical Path: The path with the longest computation time among all paths that contain zero delays.
- Loop: Directed path that begins and ends at the same node.
- Loop Bound = $t_l/w_l$, where $t_l$ is the loop computation time and $w_l$ is the number of delays in the loop.
- Critical Loop: the loops in which has maximum loop bound.
- Iteration Bound: maximum loop bound, i.e., a fundamental limit for recursive algorithms.

Loop bounds: $4/2 \text{ u.t. (max)}, \ 5/3 \text{ u.t.}, \ 5/4 \text{ u.t.}$
Iteration bound

Definition: \( T_\infty = \max \left\{ \frac{t_l}{w_l} \right\} \)

(a) Iteration bound = \( \frac{6}{2} = 3 \) u.t.

(b) Iteration bound = Max \{ \( \frac{6}{2} \), \( \frac{11}{1} \) \} = 11 u.t.
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Longest Path Matrix (LPM) Algorithm

- A series of matrix is constructed, and the iteration bound is found by examining the diagonal elements of the matrices.

- $d : \# \text{ of delays}$

- Compute $L^{(1)} \sim L^{(m)}$, $m=1, 2, 3, \ldots, d$

- $l_{i,j}^{(m)}$ : The longest computation time of all paths from delay element $d_i$ to delay element $d_j$ that pass through exactly $m-1$ delays, where the delay $d_i$ and delay $d_j$ are not included for $m-1$ delays.

- If no path exists, then the value of $l_{i,j}^{(m)}$ equals -1.
A DFG with Three Loops Using LPM

(1/3)

$$\mathbf{L}^{(1)} = \begin{bmatrix}
-1 & 0 & -1 & -1 \\
4 & -1 & 0 & -1 \\
5 & -1 & -1 & 0 \\
5 & -1 & -1 & -1 \\
\end{bmatrix}$$
A DFG with Three Loops Using LPM

\[
l^{(m+1)}_{i,j} = \max_{k \in K} (-1, l^{(1)}_{i,k} + l^{(m)}_{k,j})
\]

\[
L^{(2)} = \begin{bmatrix}
-1 & 0 & -1 & -1 \\
4 & -1 & 0 & -1 \\
5 & -1 & -1 & 0 \\
5 & -1 & -1 & -1 \\
\end{bmatrix}
\times
\begin{bmatrix}
-1 & 0 & -1 & -1 \\
4 & -1 & 0 & -1 \\
5 & -1 & -1 & 0 \\
5 & -1 & -1 & -1 \\
\end{bmatrix}
\]
A DFG with Three Loops Using LPM (3/3)

\[ L^{(3)} = \begin{bmatrix} 5 & 4 & -1 & 0 \\ 8 & 5 & 4 & -1 \\ 9 & 5 & 5 & -1 \\ 9 & -1 & 5 & -1 \end{bmatrix} \]

\[ L^{(4)} = \begin{bmatrix} 8 & 5 & 4 & -1 \\ 9 & 8 & 5 & 4 \\ 10 & 9 & 5 & 5 \\ 10 & 9 & -1 & 5 \end{bmatrix} \]

\[ T_\infty = \max_{i,m \in \{1,2,\ldots,d\}} \left\{ \frac{l^{(m)}_{i,i}}{m} \right\} \]

\[ T_\infty = \max_{i,m \in \{1,2,\ldots,d\}} \left\{ \frac{4}{2}, \frac{4}{2}, \frac{5}{3}, \frac{5}{3}, \frac{5}{3}, \frac{8}{4}, \frac{8}{4}, \frac{5}{4}, \frac{5}{4} \right\} = 2 \]
A Filter Using LPM

\[
\mathbf{L}^{(1)} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix} \quad \mathbf{L}^{(2)} = \begin{bmatrix} 12 & 12 \\ 16 & 16 \end{bmatrix}
\]

\[
T_\infty = \max\left\{ \frac{4}{1}, \frac{8}{1}, \frac{12}{2}, \frac{16}{2} \right\} = 8
\]
Minimum Cycle Mean (MCM) Method

1. Construct the new graph $G_d$ and $\overline{G_d}$
   - Transform from original DFG G
   - Decide the # of nodes from the # of delays in G.
   - Decide the weight of each edge

2. Compute the minimum cycle mean
   - Construct the series of $d+1$ vectors $f^{(m)}$, $m=0, 1, 2, \ldots, d$
     - An arbitrary reference node is chosen in $\overline{G_d}$ (called this node $s$). The initial vector $f^{(0)}$ is formed by setting $f^{(0)}(s)=0$ and setting the remaining nodes of $f^{(0)}$ to infinity.
   - find the min cycle mean

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VLSI-DSP-2-14
**A DFG with Three Loops Using MCM (1/4)**

- **Cycle mean**: Average length of the edge in \( c \) (Cycle = Loop)

- **Longest path length**
  - path that passes through no delays
  - longest: two loops that contain \( D_a \) and \( D_b \)
    - \( \max \{ 6, 4 \} = 6 \)
  - cycle mean = \( 6/2 = 3 \)
delay => node
longest path length (computation time) => weight $w(i,j)$

- If no zero-delay path exists from delay $d_i$ to delay $d_j$, then the edge $i \rightarrow j$ does not exist in $G_d$. 
A DFG with Three Loops Using MCM (3/4)

we will find $d+1$ vectors, $f^{(m)}$, $m=0,1,\ldots, d$

$$f^{(0)} = \begin{bmatrix} 0 \\ \infty \\ \infty \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} \infty \\ 0 \\ \infty \\ \infty \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -4 \\ \infty \\ 0 \\ \infty \end{bmatrix} \quad f^{(3)} = \begin{bmatrix} -5 \\ -4 \\ \infty \\ 0 \end{bmatrix} \quad f^{(4)} = \begin{bmatrix} -8 \\ -5 \\ -4 \\ \infty \end{bmatrix}$$

$$f^{(m)}(j) = \min_{i \in I} \left( f^{(m-1)}(i) + \bar{w}(i, j) \right)$$
A DFG with Three Loops Using MCM (4/4)

\[ T_\infty = - \min_{i \in \{1,2,\ldots,d\}} \left( \max_{m \in \{0,1,2,\ldots,d-1\}} \left( \frac{f^{(d)}(i) - f^{(m)}(i)}{d - m} \right) \right) \]

<table>
<thead>
<tr>
<th>( m )</th>
<th>( m = 0 )</th>
<th>( m = 1 )</th>
<th>( m = 2 )</th>
<th>( m = 3 )</th>
<th>( \max_{0 \leq m \leq 3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i = 1 )</td>
<td>-2</td>
<td>-( \infty )</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>-( \infty )</td>
<td>-5/3</td>
<td>-( \infty )</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>-( \infty )</td>
<td>-( \infty )</td>
<td>-2</td>
<td>-( \infty )</td>
<td>-2</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>( \infty )</td>
<td>-( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
<td>( \infty )</td>
</tr>
</tbody>
</table>

\[ T_\infty = - \min \{-2,-1,-2,\infty\} = 2 \]
A Filter Using MCM

\[ T_\infty = -\min\{-8, -6\} = 8 \]

<table>
<thead>
<tr>
<th>( i )</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 0 )</td>
<td>-12/2</td>
<td>-8/1</td>
</tr>
<tr>
<td>( m = 1 )</td>
<td>-6</td>
<td></td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>-(\infty)</td>
<td>-8/1</td>
</tr>
<tr>
<td>( m )</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

\[ f^{(0)} = \begin{bmatrix} 0 \\ \infty \end{bmatrix} \]

\[ f^{(1)} = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \]

\[ f^{(2)} = \begin{bmatrix} -12 \\ -12 \end{bmatrix} \]
Conclusion

- When the DFG is recursive, the iteration bound is the fundamental limit on the minimum sample period of a hardware implementation of the DSP program.

- Two algorithms to compute iteration bound, LPM and MCM, were explored.