Algorithm Strength Reduction in Filters and Transforms [Part I]

Recursive DFT/IDFT Design: Algorithm and Architecture

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Outline

- Introduction
- Review of First- and Second-Order Recursive Discrete Fourier Transform (DFT) Structures
  - New Recursive DFT/IDFT Algorithm and Architecture
  - Folded-Type DFT/IDFT Architecture and Implementation
- Comparison Results
- Conclusions
Orthogonal Matrix Review

- Def: Orthogonal and unitary matrices
  - Orthogonal
  \[ A^{-1} = A^T \]
  \[ A^T A = AA^T = I \]
  - Unitary
  \[ A^{-1} = A^* \]
  \[ A^* A = AA^* = I \]

- Def: If the transform matrix is an orthogonal matrix, then the transform is called an orthogonal transform.
  - Forward transform
    \[ z = WxW^T \]
  - Inverse transform
    \[ x = W^TzW = W^TWxW^TW \]

- The original signal can be reconstructed from its transformation.
Why Orthogonal Transformation?

- Energy conservation
- Energy compaction
  - Most unitary transforms tend to pack a large fraction of the average energy of a signal into a relatively few components of the transform coefficients.
- Decorrelation
  - When a signal is highly correlated, the transform coefficients tend to be uncorrected (or less correlated).
- Information preservation
  - The information carried by a signal is preserved under a unitary transform.
Orthogonal Transform Category

- Recursive-Algorithm Based Architecture
  - Advantage: less area
  - Application: dual tone multi-frequency (DTMF) Detection

- Butterfly-Based Architecture

- ROM-Based Operation Structure

- Multiplier-Accumulator Based Structure

- Hybrid of the Above

- Wavelet Transform
Recursive Algorithms for Orthogonal Transform

- Goertzel algorithm
- C-S’s algorithm
- Chebyshev polynomials
- Clenshaw’s recurrence formula (CRF)
Recursive Discrete Fourier Transform (DFT)

\[ X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn} \] \hspace{1cm} \text{---(1)}

where \( W_N = e^{\frac{j2\pi}{N}} \) and \( x[n] \) as well as \( X[k] \) denote DFT input and output sequences, respectively.

\[ X[k] = W_N^{-kN} \sum_{r=0}^{N-1} x[r] W_N^{kr} = \sum_{r=0}^{N-1} x[r] W_N^{-k(N-r)} \] \hspace{1cm} \text{---(2)}

\[ y_k[n] = \sum_{r=-\infty}^{\infty} x[r] W_N^{-k(n-r)} u[n - r] \] \hspace{1cm} \text{---(3)}

\[ X[k] = y_k[n] \bigg|_{n=N} \] \hspace{1cm} \text{---(4)}
Block Diagram of the First-Order Recursive DFT Structure

\[ H_k(z) = \frac{1}{1 - W_N^{-k} z^{-1}} \]  

\[ x[n] \xrightarrow{\sum} z^{-1} \xrightarrow{W_N^{-k}} X[k] \]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>First-Order DFT/IDFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Period</td>
<td>( T_m + 2T_a )</td>
</tr>
<tr>
<td># of Real Multipliers</td>
<td>4</td>
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<tr>
<td># of Real Multiplications for Each Output ( X[k] ) or ( x[n] )</td>
<td>( 4N )</td>
</tr>
<tr>
<td># of Real Additions for Each Output ( X[k] ) or ( x[n] )</td>
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</tr>
<tr>
<td># of Computation Cycles for N-Point DFT/IDFT</td>
<td>( N^2 )</td>
</tr>
</tbody>
</table>
Multiplexer-Type Dash-Line Implementation with Down-Sampling Value of $N$
Recursive Discrete Fourier Transform (DFT)

\[ H_k(z) = \frac{1}{1 - W_N^{-k} z^{-1}} \]

\[ = \frac{1 - W_N^k z^{-1}}{(1 - W_N^{-k} z^{-1})(1 - W_N^k z^{-1})} \]

\[ = \frac{1 - W_N^k z^{-1}}{1 - 2 \cos(2\pi k / N) z^{-1} + z^{-2}} \quad --- (6) \]

\[ H_k(z) = \frac{1 - \cos(2\pi k / N) z^{-1} + j \sin(2\pi k / N) z^{-1}}{1 - 2 \cos(2\pi k / N) z^{-1} + z^{-2}} \quad --- (7) \]
Block Diagram of the Second-Order Recursive DFT Structure

\[ H_k(z) = \frac{1 - W_N^k z^{-1}}{1 - 2\cos\left(\frac{2\pi k}{N}\right) z^{-1} + z^{-2}} \tag{6} \]

- **Parameters**
- **Second-Order DFT/IDFT**
  - Critical Period: \( T_m + 3T_a \)
  - # of Real Multipliers: 6
  - # of Real Multiplications for Each Output \( X[K] \) or \( x[n] \): \( 2N + 4 \)
  - # of Real Additions for Each Output \( X[K] \) or \( x[n] \): \( 4N + 4 \)
  - # of Computation Cycles for N-Point DFT/IDFT: \( N^2 \)
Outline

- Introduction
- Review of First- and Second-Order Recursive Discrete Fourier Transform (DFT) Structures
- New Recursive DFT/IDFT Algorithm and Architecture
- Folded-Type DFT/IDFT Architecture and Implementation
- Comparison Results
- Conclusions
Proposed Area-Efficient Recursive DFT Architecture

\[ H_k(z) = \frac{1 - \cos(2\pi k / N) z^{-1} + j \sin(2\pi k / N) z^{-1}}{1 - 2 \cos(2\pi k / N) z^{-1} + z^{-2}} \]  
---(7)
## Combinations of Register-Splitting Structures

<table>
<thead>
<tr>
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<th># of Delays</th>
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<tr>
<td>01</td>
<td>$T_m + 3T_a$</td>
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<tr>
<td>10</td>
<td>$T_m + 2T_a$</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>$T_m + 2T_a$</td>
<td>4</td>
</tr>
</tbody>
</table>

**Optimized Value**
Proposed High-Speed Area-Efficient Recursive DFT Architecture

\[ x[n] \rightarrow \sum \rightarrow \frac{1}{z^{-1}} \rightarrow \sum \rightarrow \frac{1}{z^{-1}} \rightarrow X[k] \]

- Critical Period: \( T_m + 2T_a \)
- # of Real Multipliers: 4
- # of Real Multiplications for Each Output \( X[K] \) or \( x[n] \): \( 2N + 2 \)
- # of Real Additions for Each Output \( X[K] \) or \( x[n] \): \( 4N + 4 \)
- # of Computation Cycles for N-Point DFT/IDFT: \( N^2 \)
Proposed High-Speed Area-Efficient Recursive IDFT Architecture

\[ H_n(z) = \frac{1 - \cos(2\pi n / N)z^{-1} - j\sin(2\pi n / N)z^{-1}}{1 - 2\cos(2\pi n / N)z^{-1} + z^{-2}} \]  ---(8)

\[ X[k] \rightarrow \Sigma \rightarrow z^{-1} \rightarrow \sum \rightarrow HS \leftrightarrow \rightarrow \cos(\frac{2\pi n}{N}) \rightarrow j\sin(\frac{2\pi n}{N}) \rightarrow \Sigma \rightarrow \Sigma \rightarrow z^{-1} \rightarrow \Sigma \rightarrow x[n] \]
Parallel High-Speed Area-Efficient Recursive DFT Architecture

- Parameters for Proposed Work 2 (Parallel Type)
  - Critical Period: \( T_m + 2T_a \)
  - # of Real Multipliers: \( 4N \)
  - # of Real Multiplications for Each Output \( x[k] \) or \( x[n] \): \( 2N + 2 \)
  - # of Real Additions for Each Output \( x[k] \) or \( x[n] \): \( 4N + 4 \)
  - # of Computation Cycles for \( N \)-Point DFT/IDFT: \( 2N \)
Folded High-Speed Area-Efficient Recursive DFT Architecture

Parameters

<table>
<thead>
<tr>
<th>Proposed Work 3 (Folded Type)</th>
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</thead>
<tbody>
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<td># of Real Multipliers</td>
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<tr>
<td># of Real Multiplications</td>
</tr>
<tr>
<td># of Real Additions</td>
</tr>
<tr>
<td># of Computation Cycles</td>
</tr>
</tbody>
</table>
New Recursive Formula for DFT (1/5)

\[ y[k] = \sum_{n=0}^{N-1} x[n] \cdot W_N^{kn} \] , where \( W_N = e^{-j2\pi/N} \)

\[ y[k] = \sum_{n=0}^{N/2-1} x'[n] \cdot W_N^{kn} + \sum_{n=0}^{N/2-1} x'[N-1-n] \cdot W_N^{k(N-1-n)} \]

\[ = \sum_{n=0}^{N/2-1} \left( x'[n] + W_N^{-k} \cdot x'[N-1-n] \right) \cdot \cos\left( \frac{2\pi kn}{N} \right) \]

\[ + j \sum_{n=0}^{N/2-1} \left( -x'[n] + W_N^{-k} \cdot x'[N-1-n] \right) \cdot \sin\left( \frac{2\pi kn}{N} \right) = y_{DCT}[k] + jy_{DST}[k] \]

, where

\[ y_{DCT}[k] = \sum_{n=0}^{N/2-1} \left( x'[n] + W_N^{-k} \cdot x'[N-1-n] \right) \cdot \cos\left( \frac{2\pi kn}{N} \right) \]

\[ y_{DST}[k] = - \sum_{n=0}^{N/2-1} \left( x'[n] - W_N^{-k} \cdot x'[N-1-n] \right) \cdot \sin\left( \frac{2\pi kn}{N} \right) \]

Assume

\[ r_k[n] = x'[n] + W_N^{-k} \cdot x'[N-1-n] \]

\[ s_k[n] = x'[n] - W_N^{-k} \cdot x'[N-1-n] \]

\[ \text{VLSI-DSP-9-19} \]
New Recursive Formula for DFT (2/5)

For the DCT, replacing “n” by “N/2 – 1 – n”

\[ y_{DCT}[k] = \sum_{n=0}^{N/2-1} r_k[n] \cdot \cos\left(\frac{2\pi kn}{N}\right) \]

\[ = \sum_{n=0}^{N/2-1} r_k[N/2 - 1 - n] \cdot \cos\left(\frac{2\pi k(N/2 - 1 - n)}{N}\right) \]

\[ = (-1)^k \sum_{n=0}^{N/2-1} r_k[N/2 - 1 - n] \cdot \cos\left(\frac{2\pi k(n+1)}{N}\right) \]

\[ = (-1)^k \cdot g_{N/2-1}(k) \]

, where

\[ g_{N/2-1}(k) = \sum_{n=0}^{N/2-1} r_k[N/2 - 1 - n] \cdot \cos\left(\frac{2\pi k(n+1)}{N}\right) \]

\[ g_i(k) = \sum_{n=0}^{i} r_k(i - n) \cdot \cos((n + 1)\theta_k) \]

\[ \theta_k = \frac{2\pi k}{N} \]
New Recursive Formula for DFT (3/5)

Applying the Chebyshev polynomials:

\[
\cos(r\theta) = 2\cos((r-1)\theta) \cdot \cos\theta - \cos((r-2)\theta)
\]

\[
g_i(k) = \sum_{n=0}^{i} r_k[i-n] \cdot \left\{2\cos(n\theta_k) \cdot \cos\theta_k - \cos((n-1)\theta_k)\right\}
\]

\[
= 2\sum_{n=0}^{i} r_k[i-n] \cdot \cos(n\theta_k) \cdot \cos\theta_k - \sum_{n=0}^{i} r_k[i-n] \cdot \cos((n-1)\theta_k)
\]

\[
= 2r_k[i] \cdot \cos\theta_k + 2\sum_{n=0}^{i-1} r_k[i-1-n] \cdot \cos((n+1)\theta_k) \cdot \cos\theta_k
\]

\[-r_k[i] \cdot \cos\theta_k - r_k[i-1] - \sum_{n=0}^{i-2} r_k[i-2-n] \cdot \cos((n+1)\theta_k)
\]

\[
= r_k[i] \cdot \cos\theta_k - r_k[i-1] + 2\cos\theta_k \cdot g_{i-1}(k) - g_{i-2}(k)
\]
New Recursive Formula for DFT (4/5)

For the DST, replacing “n” by “N/2 – 1 – n”

\[ y_{DST}[k] = - \sum_{n=0}^{N/2-1} s_k[n] \cdot \sin\left(\frac{2\pi k n}{N}\right) \]

\[ = - \sum_{n=0}^{N/2-1} s_k[N/2 - 1 - n] \cdot \sin\left(\frac{2\pi k (N/2 - 1 - n)}{N}\right) \]

\[ = (-1)^k \sum_{n=0}^{N/2-1} s_k[N/2 - 1 - n] \cdot \sin\left(\frac{2\pi k (n+1)}{N}\right) \]

\[ = (-1)^k \cdot \sum_{n=0}^{N/2-1} s_k[N/2 - 1 - n] \cdot \sin((n + 1)\theta_k) \]

\[ = (-1)^k \cdot h_{N/2-1}(k) \]

where \[ h_{N/2-1}(k) = \sum_{n=0}^{N/2-1} s_k[N/2 - 1 - n] \cdot \sin((n + 1)\theta_k). \]
New Recursive Formula for DFT (5/5)

Applying the Chebyshev polynomials:

\[
\sin(r\theta) = 2\sin((r-1)\theta) \cdot \cos\theta - \sin((r-2)\theta)
\]

\[
h_j(k) = \sum_{n=0}^{j} s_k [j-n] \cdot \sin((n+1)\theta_k)
\]

\[
= \sum_{n=0}^{j} s_k [j-n] \cdot \{2 \sin(n\theta_k) \cdot \cos\theta_k - \sin((n-1)\theta_k)\}
\]

\[
= 2 \sum_{n=0}^{j} s_k [j-n] \cdot \sin(n\theta_k) \cdot \cos\theta_k - \sum_{n=0}^{j} s_k (j-n) \cdot \sin((n-1)\theta_k)
\]

\[
= 2 \sum_{n=0}^{j-1} s_k [j-1-n] \cdot \sin((n+1)\theta_k) \cdot \cos\theta_k + s_k [j] \cdot \sin\theta_k
\]

\[
- \sum_{n=0}^{j-2} s_k [j-2-n] \cdot \sin((n+1)\theta_k)
\]

\[
= s_k [j] \cdot \sin\theta_k + 2 \cos\theta_k \cdot h_{j-1}(k) - h_{j-2}(k)
\]
The Proposed Recursive DFT formula:

\[
g(k, z) = \frac{\cos \theta_k - z^{-1}}{1 - 2 \cos \theta_k z^{-1} + z^{-2}}
\]

\[
r_k(z) = \frac{\cos \theta_k - z^{-1}}{1 - 2 \cos \theta_k z^{-1} + z^{-2}}
\]

\[
h(k, z) = \frac{\sin \theta_k}{1 - 2 \cos \theta_k z^{-1} + z^{-2}}
\]

\[
s_k(z) = \frac{\sin \theta_k}{1 - 2 \cos \theta_k z^{-1} + z^{-2}}
\]

The Proposed Recursive IDFT formula:

\[
g(n, z) = \frac{\cos \theta_n - z^{-1}}{1 - 2 \cos \theta_n z^{-1} + z^{-2}}
\]

\[
r_n(z) = \frac{\cos \theta_n - z^{-1}}{1 - 2 \cos \theta_n z^{-1} + z^{-2}}
\]

\[
h(n, z) = \frac{\sin \theta_n}{1 - 2 \cos \theta_n z^{-1} + z^{-2}}
\]

\[
s_n(z) = \frac{\sin \theta_n}{1 - 2 \cos \theta_n z^{-1} + z^{-2}}
\]
Block Diagram of Low Computation Cycle for DCT part and DST Part

\[ r_k[n] \rightarrow \sum \rightarrow Z^{-1} \rightarrow \cos \theta_k \rightarrow \sum \rightarrow (-1)^k y_{DCT}[k] \]

\[ (-1)^k y_{DST}[k] \]

\[ s_k[n] \rightarrow \sum \rightarrow Z^{-1} \rightarrow \sin \theta_k \rightarrow \sum \rightarrow 2 \cos \theta_k \rightarrow Z^{-1} \]

\[ Z^{-1} \]

\[ 0 \]

\[ 0 \]
Resource Sharing and Register Splitting

- **Computation Sharing**
  - Common multiplier for \( \cos \theta_K \)

- **Register Splitting**
  - Reduce critical path
  - Notations 0 and 1: delay elements
  - With the minimum critical path: 10
  - With the fewest registers: 10
  - \( \lll \) : one-bit left shift
  - \( \lll \) : the forward pipeline register

<table>
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<th>Com.</th>
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<th>10</th>
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<td>( 2T_m + 3T_a )</td>
<td>( T_m + 2T_a )</td>
<td>( T_m + 2T_a )</td>
</tr>
<tr>
<td># of Reg.</td>
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<td>2</td>
<td>2</td>
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<tr>
<td>Opt.</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
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</table>
Low Computation Cycle and High-Throughput DFT Architecture

\[ r_k[n] \rightarrow \sum \rightarrow \cos \theta_k \]

\[ s_k[n] \rightarrow \sum \rightarrow \sin \theta_k \]

\[ 6 \text{ Real Multipliers} \]

Critical Period: \( T_m + 2T_a \)
New Recursive Formula for IDFT (1/5)

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N-1} y[k] \cdot W_N^{-kn} \] , where \[ W_N = e^{-j2\pi/N} \]

\[ x[n] = \frac{1}{N} \sum_{k=0}^{N/2-1} y'[k] \cdot W_N^{-kn} + \frac{1}{N} \sum_{k=0}^{N/2-1} y'[N-1-k] \cdot W_N^{(k+1)n} \]

\[ = \frac{1}{N} \sum_{k=0}^{N/2-1} \left( y'[k] + W_N^n \cdot y'[N-1-k] \right) \cdot \cos\left( \frac{2\pi kn}{N} \right) \]

\[ + j \cdot \frac{1}{N} \sum_{k=0}^{N/2-1} \left( y'[k] - W_N^n \cdot y'[N-1-k] \right) \cdot \sin\left( \frac{2\pi kn}{N} \right) = x_{IDCT}[n] + jx_{IDST}[n] \]

, where \[ x_{IDCT}[n] = \frac{1}{N} \sum_{k=0}^{N/2-1} \left( y'[k] + W_N^n \cdot y'[N-1-k] \right) \cdot \cos\left( \frac{2\pi kn}{N} \right) \]

\[ x_{IDST}[n] = \frac{1}{N} \sum_{k=0}^{N/2-1} \left( y'[k] - W_N^n \cdot y'[N-1-k] \right) \cdot \sin\left( \frac{2\pi kn}{N} \right) \]

Assume \[ r_n[k] = y'[k] + W_N^n \cdot y'[N-1-k] \]

\[ s_n[k] = y'[k] - W_N^n \cdot y'[N-1-k] \]
New Recursive Formula for IDFT (2/5)

For the IDCT, replacing “n” by “N/2 − 1 − n”

\[ x_{IDCT}[n] = \frac{1}{N} \sum_{k=0}^{N/2-1} \left( y'[k] + W_N^n \cdot y'[N − 1 − k] \right) \cdot \cos\left(\frac{2\pi kn}{N}\right) \]

\[ = \frac{1}{N} \sum_{k=0}^{N/2-1} r_n[N/2 − 1 − k] \cdot \cos\left(\frac{2\pi n(N/2 − 1 − k)}{N}\right) \]

\[ = \frac{(-1)^n}{N} \sum_{k=0}^{N/2-1} r_n[N/2 − 1 − k] \cdot \cos\left(\frac{2\pi n(k + 1)}{N}\right) \]

\[ = \frac{(-1)^n}{N} \cdot g_{N/2-1}(n) \]

where

\[ g_{N/2-1}(n) = \sum_{k=0}^{N/2-1} r_n[N/2 − 1 − k] \cdot \cos\left(\frac{2\pi n(k + 1)}{N}\right) \]

\[ g_i(n) = \sum_{k=0}^{i} r_n[i − k] \cdot \cos((k + 1)\theta_n) \] \[ \theta_n = \frac{2\pi n}{N} \]
New Recursive Formula for IDFT (3/5)

Applying the Chebyshev polynomials:

\[ \cos(r\theta) = 2\cos((r-1)\theta) \cdot \cos\theta - \cos((r-2)\theta) \]

\[ g_i[n] = \sum_{k=0}^{i} r_n[i-k] \cdot \{2\cos(k\theta_n) \cdot \cos\theta_n - \cos((k-1)\theta_n)\} \]

\[ = r_n[i] \cdot \cos\theta_n - r_n[i-1] + 2\cos\theta_n \cdot g_{i-1}(n) - g_{i-2}(n) \]

\[ \frac{g(n, z)}{r_n(z)} = \frac{\cos\theta_n - z^{-1}}{1 - 2\cos\theta_n z^{-1} + z^{-2}} \]
New Recursive Formula for IDFT (4/5)

For the IDST, replacing “n” by “N/2 – 1 – n”

\[ x_{IDST}[n] = \frac{1}{N} \sum_{k=0}^{N/2-1} s_n[k] \cdot \sin\left(\frac{2\pi kn}{N}\right) \]

\[ = \frac{1}{N} \sum_{k=0}^{N/2-1} s_n[N/2 - 1 - k] \cdot \sin\left(\frac{2\pi n(N/2 - 1 - k)}{N}\right) \]

\[ = -(-1)^n \sum_{k=0}^{N/2-1} s_n[N/2 - 1 - k] \cdot \sin((k + 1)\theta_n) \]

\[ = -(-1)^n \cdot h_{N/2-1}(n) \]

, where \[ h_{N/2-1}(n) = \sum_{k=0}^{N/2-1} s_n[N/2 - 1 - k] \cdot \sin((k + 1)\theta_n) \]
New Recursive Formula for IDFT (5/5)

Applying the Chebyshev polynomials:

\[ \sin(r\theta) = 2\sin((r-1)\theta) \cdot \cos \theta - \sin((r-2)\theta) \]

\[ h_j(n) = \sum_{k=0}^{j} s_n[j-k] \cdot \sin((k+1)\theta_n) \]

\[ = \sum_{k=0}^{j} s_n[j-k] \cdot \{2\sin(k\theta_n) \cdot \cos \theta_n - \sin((k-1)\theta_n)\} \]

\[ = s_n[j] \cdot \sin \theta_n + 2\cos \theta_n \cdot h_{j-1}(n) - h_{j-2}(n) \]

\[ \frac{h(n, z)}{s_n(z)} = \frac{\sin \theta_n}{1 - 2\cos \theta_n z^{-1} + z^{-2}} \]
Low Computation Cycle and High-Throughput Recursive IDFT Architecture

- 6 Real Multipliers
- Critical Period: $T_m + 2T_a$
Folded-Type DFT/IDFT Architecture

- **Data Buffer**
  - 64 16-bit word length complex data storage
- **Control Unit**
  - Clock Gated Control
  - Sequence Controller
  - Parameter Controller
- **32 PEs**
  - Pre-processing for $S_k$ and $r_k$
  - TWO PEs for DST and DCT

**Diagram**

- **Input Unit**
  - $x'[n]$ and $x'[N-1-n]$
- **Pre-processing**
  - $W_N^k$
- **Recursive PE**
  - $r_k[n]$, $s_k[n]$, $PE_{lm}$, and $PE_{Re}$
- **RDFT Unit**
  - (1), (2), (3)
- **Output Unit**
  - $y[k]$

**System Components**

- **Data Buffer**
- **Control Unit**
- **32 PEs**
- **Clock Start**
- **Reset**

**Notes**

- **Folded-Type DFT/IDFT Architecture**
- **Steps**
  - Pre-processing
  - Recursive processing
- **Variables**
  - $x[n]$, $x'[n]$, $x'[N-1-n]$, $r_k[n]$, $s_k[n]$
Folded-Type DFT/IDFT Architecture

- Regularly constructed by N/2 PE’s in parallel
- No intermediate register bank needed
- Further reduce the computation cycle to
  \[ N = \frac{N^2/2}{N/2} \]
- Processor latency: \( N \) clock
  (Computation cycles)
- Critical Path: \( T_m + 2T_a \)
Simulation Result

- **IFFT**: the signal $x(t)$ pass through floating point IFFT block
- **AWGN**: channel model
- **FFT**: The test modules include four different systems: butterfly, RDFT, Goertzel 1st and 2st architecture.
Three Phases for IC Implementation

**Description**
Specify and capture the ideal into some formal representations

**Verification**
Verify the correctness of design and implementation

**Implementation**
Refine the design through all phases
Chip Layout

- TSMC 0.13 um CMOS process
- 64 point DFT
- 16-bit input word length
- 47.67 ns @1.3V
- Chip area: 1822 um x 1822 um
- Power Consumption: 29.94 mW@20 MHz
Outline

• Introduction
• Review of First- and Second-Order Recursive Discrete Fourier Transform (DFT) Structures
• New Recursive DFT/IDFT Algorithm and Architecture
• Folded-Type DFT/IDFT Architecture and Implementation
  • Comparison Results
  • Conclusions
## Comparison Results (1/2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>First-Order DFT/IDFT</th>
<th>Second-Order DFT/IDFT</th>
<th>Proposed Work 1 (Core Type)</th>
<th>Proposed Work 2 (Parallel Type)</th>
<th>Proposed Work 3 (Folded Type)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical Period</td>
<td>$T_m + 2T_a$</td>
<td>$T_m + 3T_a$</td>
<td>$T_m + 2T_a$</td>
<td>$T_m + 2T_a$</td>
<td>$T_m + 2T_a$</td>
</tr>
<tr>
<td># of Real Multipliers</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>$4N$</td>
<td>$2N$</td>
</tr>
<tr>
<td># of Real Multiplications for Each Output $X[K]$ or $x[n]$</td>
<td>$4N$</td>
<td>$2N + 4$</td>
<td>$2N + 2$</td>
<td>$2N + 2$</td>
<td>$2N + 2$</td>
</tr>
<tr>
<td># of Real Additions for Each Output $X[K]$ or $x[n]$</td>
<td>$4N$</td>
<td>$4N + 4$</td>
<td>$4N + 4$</td>
<td>$4N + 4$</td>
<td>$4N + 4$</td>
</tr>
<tr>
<td># of Computation Cycles for N-Point DFT/IDFT</td>
<td>$N^2$</td>
<td>$N^2$</td>
<td>$N^2$</td>
<td>$2N$</td>
<td>$3N$</td>
</tr>
</tbody>
</table>
## Comparison Results (2/2)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Second Order DFT/IDFT</th>
<th>V-Y’s Structure [10] (Core Type)</th>
<th>Y-C’s Structure [5] (FFR-DFT)</th>
<th>Proposed Work 1 (Core Type)</th>
<th>Proposed Work 2 (Folded Type)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong># of Computation Cycles for Each ( y[k] ) or ( x[n] )</strong></td>
<td>( N )</td>
<td>( N )</td>
<td>( N )</td>
<td>( N/2 )</td>
<td>( N/2 )</td>
</tr>
<tr>
<td><strong># of Computation Cycles for N-Point DFT/IDFT</strong></td>
<td>( N^2 )</td>
<td>( N^2 )</td>
<td>( N^2 )</td>
<td>( N^2/2 )</td>
<td>( N )</td>
</tr>
<tr>
<td><strong>Clock Period</strong></td>
<td>( T_m + 3T_a )</td>
<td>( T_m + 2T_a )</td>
<td>( 2T_m + 5T_a )</td>
<td>( T_m + 2T_a )</td>
<td>( T_m + 2T_a )</td>
</tr>
<tr>
<td><strong># of Real Multipliers</strong></td>
<td>6</td>
<td>4</td>
<td>6</td>
<td>6</td>
<td>3N</td>
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<tr>
<td></td>
<td>(Pre-processing Excluded)</td>
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<td>(Pre-processing Excluded)</td>
<td>(Pre-processing Excluded)</td>
</tr>
</tbody>
</table>
Conclusions

Three new recursive DFT/IDFT architectures based on Goertzel algorithm by the hybrid of module-sharing and register-splitting schemes
  - High-speed due to register-splitting scheme
  - Low-area due to module-sharing scheme

Propose a lower computation cycle and high-throughput DFT/IDFT algorithm and architecture
  - Low computation cycle due to input strength reduction scheme
  - High-throughput due to register-splitting scheme
References

References (Cont’d)


References (Cont’d)